Available online at www.sciencedirect.com
SCIENCE DIRECT
ACADEMIC PRESS
Journal of Sound and Vibration 266 (2003) 33–48
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Dependence of the induced loss factor on the coupling forms and coupling strengths: energy analysis

G. Maidanik*, K.J. Becker

Naval Surface Warfare Center NSWCCD (DTMB) Carderock Division, 9500 MacArthur Blvd. West Bethesda, MD 20817-5700, USA

Received 7 May 2001; accepted 2 September 2002

Abstract

In two recent papers, the induced loss factor is determined via the modification to the loss factor in the linear impedance of a master oscillator caused by its coupling to a set of satellite oscillators. A loss factor is basically an energetic quantity and, therefore, one may inquire whether the induced loss factor may be estimated via an energy analysis (EA). An answer to this question is sought. It is shown that the linear impedance analysis and EA yields identical results for the induced loss factor in the appropriate frequency range. This frequency range spans the distribution of resonance frequencies of the satellite oscillators. In this frequency range, the identity of the results is not only in terms of gross features but also in detail. Finally, the relationship of EA to statistical energy analysis (SEA) is explored. The loss factors assigned to the satellite oscillators are cast in terms of modal overlap parameters. It is found necessary for the validation of SEA, in the light of EA, that these parameters exceed a certain threshold. In specific situations of interest the threshold values may exceed unity.

1. Introduction

In two recent papers, the influence on the response behavior of a master oscillator due to its coupling to a set of satellite oscillators is derived and examined [1,2]. The complex, comprising a master oscillator coupled to a set of satellite oscillators, is sketched in Fig. 1. The examination is conducted in terms of the induced loss factor as a function of the normalized frequency variable. The normalizing frequency is the resonance frequency of the master oscillator in isolation. The induced loss factor accounts for the modification in the loss factor in the linear impedance of the

*Corresponding author. Tel.: +1-301-227-1292; fax: +1-301-227-5529.
E-mail address: beckerkj@nswccd.navy.mil (G. Maidanik).

0022-460X/03/$ - see front matter
doi:10.1016/S0022-460X(02)01390-1

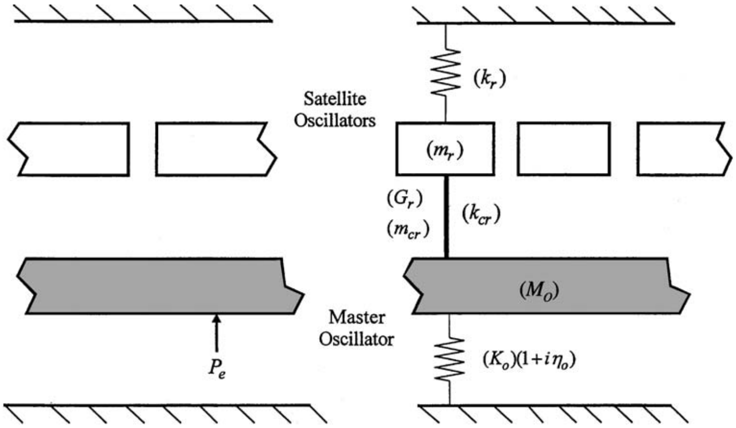

Fig. 1. A set of satellite oscillators coupled to a master oscillator.

master oscillator caused by the coupling. This modification is the focus of the investigation performed in Refs. [1,2] and is also the focus of the investigation in this paper. Here, however, the emphasis is on an energy analysis (EA), rather than on a linear impedance analysis (LIA).

As Fig. 1 indicates, the coupling between the master oscillator and a satellite oscillator allows for stiffness, mass and gyroscopic elements and for mixtures of these elemental coupling forms [3,4]. With the assistance of Fig. 1, the linear equations of motion for a master oscillator in isolation, for a coupled master oscillator and for a typical satellite oscillator are stated in forms that are the same as those obtained in Ref. [2]. For this reason Ref. [2] needs to be consulted as fundamental to the investigation conducted herein. As in Ref. [2], similarity is imposed on the stiffnesses (springs) that are placed on either side of the mass elements of a satellite oscillator. This similarity defines two spring factors and a specific distribution for the resonance frequencies of the satellite oscillators. In this distribution, the resonance frequencies are indexed sequentially with equal numbers of satellite oscillators on either side of the resonance frequency of the master oscillator. As already intimated, the resonance frequency of the master oscillator is also used to normalize frequencies [2]. In Ref. [2] the various impedances that define the complex are stated and the LIA is developed and used to determine the induced loss factor. The induced loss factor adds to the indigenous loss factor in the impedance of the master oscillator when it is coupled to the set of satellite oscillators. When couplings are absent, the induced loss factor is equal to zero. In this paper, the impedance equations of motion are use to develop an EA. The quantities in the analysis are the energies stored in the oscillators and in the couplings and the external force-drive is expressed in terms of the external power input [3]. The stored energies are normalized by the kinetic energy stored in the master oscillator and the powers by this kinetic energy times the frequency. Although the analytic procedure is different from that conducted in Ref. [2], computations of the two induced loss factors by the two procedures show them to be identical. So much so that the relevant figures in Ref. [2] are directly applicable.

Two renormalizations of the stored energies and the powers define two loss factors, respectively. The one pertains to a renormalization that is effected by the stored energy; kinetic plus potential, that is stored in the master oscillator only. The other pertains to a renormalization that is effected by the stored energy in the entire complex, therefore, including the energies stored in the satellite oscillators and in their couplings. The first loss factor is dubbed virtual and is designated illegitimate [5]. The second loss factor is dubbed effective and is designated legitimate [5]. It is argued that the difference between these two loss factors largely accounts for the energy imbalance that beset earlier investigations in this research area [6–13].

Finally, the EA, developed herein, estimates the modal coupling strength. The modal coupling strength is a measure of the ratio of the average energy stored in a single satellite oscillator and its coupling to that stored in the master oscillator [3]. Comparing mean-value modal coupling strengths, derived via the EA to that derived via the statistical energy analysis (SEA), reveals that the validity of SEA necessarily demands that the satellite oscillators be assigned loss factors with associated modal overlap parameters that exceed a determined threshold value [3,14]. Examples of computed threshold values are briefly cited.

2. Derivation of the energy equations of motion

The linear equations of motion of the complex composed of a master oscillator coupled to a set of satellite oscillators may be largely derived via the Lagrange equations. The Lagrangian describes the difference between the kinetic and potential energies stored in the oscillators and in the couplings [3]. (Notwithstanding that the linear equations of motion are correctly stated in Ref. [3], a persistent typographical error in the preceding Lagrange equations needs to be corrected.) Although the kinetic and potential energies are here determined separately, it is the energy and not the Langrangian, as such, that is of immediate interest. The kinetic energy $E_{oK}(y)$ plus the potential energy $E_{oP}(y)$ stored in the master oscillator, as functions of the normalized frequency (y), is given by

$$E_o(y) = E_{oK}(y) + E_{oP}(y), \quad E_{oK}(y) = (1/2)M_o|V_o(y)|^2,$$
$$E_{oP}(y) = (y)^{-2}E_{oK}(y), \quad y = (\omega/\omega_o), \quad \omega_o = (K_o/M_o)^{1/2}, \tag{1}$$

where $E_o(y)$ is the stored energy, $V_o(y)$ is the response, M_o is the mass element and K_o is the stiffness element in the master oscillator [3]. Analogously, the kinetic energy $E_{rK}(y)$ and the potential energy $E_{rP}(y)$ that is stored in the rth satellite oscillator is given by

$$E_r(y) = E_{rK}(y) + E_{rP}(y), \quad E_{rK}(y) = (1/2)m_r|V_r(y)|^2,$$
$$E_{rp}(y) = (z_r)^2E_{rK}(y), \quad z_r = (x_r/y), \quad x_r = (\omega_r/\omega_o), \quad (\omega_r) = (k_{ro}/m_r)^{1/2}, \tag{2}$$

where $E_r(y)$ is the stored energy, $V_r(y)$ is the response, m_r is the mass element and k_{ro} is the stiffness element of the rth satellite oscillator, respectively [3]. In addition, the kinetic energy $E_{crK}(y)$ and the potential energy $E_{crP}(y)$ stored in the coupling, between the master oscillator and the rth satellite oscillator, is given by

$$E_{cr}(y) = E_{crK}(y) + E_{crP}(y), \tag{3a}$$

$$E_{crK(y)} = (1/2)m_r\left[\bar{m}_{cr}|V_o(y) + V_r(y)|^2 + \mathrm{Im}\{(g_r/y)[V_o(y) + V_r(y)][V_o(y) - V_r(y)]^*\}\right], \tag{3b}$$

$$E_{crP} = (1/2)m_r(z_{cr})^2|V_o(y) - V_r(y)|^2, \quad z_{cr} = (x_{cr}/y),$$
$$x_{cr} = (\omega_{cr}/\omega_o), \quad \omega_{cr} = (k_{cro}/m_r)^{1/2}, \tag{3c}$$

where $E_{cr}(y)$ is the stored energy, m_{cr} is the mass element, $\bar{m}_{cr} = (m_{cr}/m_r)$, k_{cro} is the stiffness element and g_r is the normalized gyroscopic element, $g_r = [G_r/(\omega_o m_r)]$, in the coupling between the master oscillator and the rth satellite oscillator [3]. Using the impedance version of the equations of motion as derived in Refs. [1,2], Eq. (3) may be cast more explicitly in terms of the parameters that define the complex under investigation. From Eqs. (3a)–(3c) this procedure yields

$$\bar{E}_o(y) \cong [1 + (y)^{-2}], \quad \bar{E}_o(y) = [E_o(y)/E_{oK}(y)], \tag{4a}$$

$$\bar{E}_r(y) = [E_r(y)/E_{oK}(y)] = \bar{m}_r[\{1 + (z_r)^2\}|B_r|^2], \tag{4b}$$

$$\bar{E}_{cr}(y) = [E_{cr}(y)/E_{oK}(y)] = \bar{m}_r[\bar{m}_{cr}|1 + B_r|^2 + (z_{cr})^2|1 - B_r|^2 - \mathrm{i}(g_r/y)(B_r - B_r^*)], \tag{4c}$$

respectively, where a *single bar* over a stored energy quantity, e.g., $\bar{E}_r(y)$, indicates a normalization by the kinetic stored energy $E_{oK}(y)$. In Eq. (4) the quantities $B_r(y)$, z_r, z_{cr} and $\bar{m}_r$ are defined as

$$B_r(y) = -[\bar{m}_{cr} + (z_{cr})^2(1 + \mathrm{i}\eta_{cr}) - \mathrm{i}(g_r/y)][(1 + \bar{m}_{cr}) - (z_{rr})^2(1 + \mathrm{i}\eta_{rr})]^{-1}, \tag{5a}$$

$$(z_{rr})^2(1 + \mathrm{i}\eta_{rr}) = (z_r)^2(1 + \mathrm{i}\eta_r) + (z_{cr})^2(1 + \mathrm{i}\eta_{cr}), \quad \bar{m}_r = (m_r/M_o) \tag{5b}$$

where η_r and η_{cr} are the assigned loss factors associated with the stiffness elements k_{ro} and k_{cro}, respectively [2,3] [cf., Eqs. (2) and (3)]. In assessing the distribution of the stored energies, it is compelling to distinguish between the normalized energy

$$\bar{E}_o(y) = [1 + (y)^{-2}], \tag{6a}$$

that is stored in the master oscillator and the normalized energy

$$\bar{E}_{os}(y) = \sum_1^R \bar{E}_{or}(y), \quad \bar{E}_{or}(y) = \bar{E}_r(y) + \bar{E}_{cr}(y), \tag{6b}$$

that is stored in the satellite oscillators and in the couplings of these oscillators to the master oscillator [cf., Eqs. (1)–(4)]. These normalized stored energies are, clearly, functions of the normalized frequency (y), $y = (\omega/\omega_o)$ and they are sustained in the complex by the external input power $\Pi_e(y)$. This external input power is generated by the external force-drive $P_e(y)$ that acts on the master oscillator; the satellite oscillators are not externally force driven in this analysis (cf., Fig. 1). In an endeavor to determine an energy based induced loss factor one needs to determine this external input power. The endeavor is facilitated by briefly recalling the manner by which an impedance based induced loss factor is determined. From Refs. [1,2] one recalls the impedance equations of motion to be in the form

$$Z_o(\omega)V_o(\omega) = P_e(\omega) \Rightarrow Z_o(y)V_o(y) = P_e(y), \tag{7a}$$

$$Z_o(y) = (\mathrm{i}\omega M_o)[1 - (y)^{-2}\{[1 - S(y)] + \mathrm{i}[\eta_o + \eta_I(y)]\}], \tag{7b}$$

$$[S(y) - \mathrm{i}\eta_I(y)] = \sum_1^R [S_r(y) - \mathrm{i}\eta_{Ir}(y)],$$

$$\begin{aligned} S_r(y) - \mathrm{i}\eta_{Ir}(y) = (y)^2\{\bar{m}_r\{[1-(z_r)^2(1+\mathrm{i}\eta_r)] \\ \times [\bar{m}_{cr} - (z_{cr})^2(1+\eta_{cr})] - (q_{cr}/y)^2\} \\ \times [(1+\bar{m}_{cr}) - (z_{rr})^2(1+\mathrm{i}\eta_{rr})]^{-1}\}, \end{aligned} \tag{7c}$$

$$(q_{cr}/y)^2 = 4\bar{m}_{cr}(z_{cr})^2(1+\mathrm{i}\eta_{cr}) + (g_r/y)^2, \tag{7d}$$

where $Z_o(\omega)$, $V_o(\omega)$ and $P_e(\omega)$ are, respectively, the linear impedance, the response and the external force-drive that is applied to the master oscillator. This force-drive induces the response $V_o(\omega)$ in the master oscillator and the response $V_r(\omega)$ in the rth satellite oscillator when the set of satellite oscillators is coupled to the master oscillator. In Eq. (7) the parameters $\{\bar{m}_r, z_r, \eta_r, \bar{m}_{cr}, z_{cr}, \eta_{cr}, z_{rr}, \eta_{rr}, g_r\}$ define the satellite oscillators and their couplings to the master oscillator (cf., Eq. (5)). In Ref. [2], $\eta_I(y)$ is identified with the induced loss factor. As briefly depicted in Eq. (7), $\eta_I(y)$ is derived on the basis of the LIA. In the present paper, an endeavor is made to derive the induced loss factor on the basis of an EA.

3. Conservation of energy—the balance of power—and the derivation of the induced loss factor via an EA

The normalized external input power may be derived from Eq. (7) in the form

$$\begin{aligned} &\bar{\Pi}_e(y) = [\Pi_e(y)/\{\omega E_{oK}(y)\}], \quad \bar{\Pi}_e(y) = Re\{P_e(y)V_o^*(y)\}, \\ &(y^2/2)\bar{\Pi}_e(y) = [\eta_o + \eta_I(y)], V_o(\omega) \Rightarrow V_o(y), \quad P_e(\omega) \Rightarrow P_e(y), \end{aligned} \tag{8}$$

where, hereafter, a *single bar* over a power quantity, e.g., $\bar{\Pi}_e(y)$, indicates a normalization by the power quantity $\{\omega E_{oK}(y)\}$ (cf., Eq. (4)). The third of Eq. (8) states that a portion $\bar{\Pi}_o(y)$ of $\bar{\Pi}_e(y)$ is dissipated in the master oscillator such that

$$(y^2/2)\bar{\Pi}_o(y) = \eta_o^e(y), \quad \eta_o^e(y) = \eta_o, \quad \bar{\Pi}_o(y) = [\Pi_o(y)/\{\omega E_{oK}(y)\}] \tag{9a}$$

and the portion $\bar{\Pi}_{os}(y)$ of $\bar{\Pi}_e(y)$ is dissipated in the satellite oscillators and in the couplings

$$(y^2/2)\bar{\Pi}_{os}(y) = \eta_I^e(y), \quad \bar{\Pi}_{os}(y) = [\Pi_{os}(y)/\{\omega E_{oK}(y)\}], \tag{9b}$$

where, as already intimated, the superscript e indicates that the loss factors are determined via an EA and not via an LIA. Thus, $\eta_I^e(y)$ is the induced loss factor derived on the basis of the EA. The conservation of energy (the balance of power) demands that

$$\bar{\Pi}_e(y) = \bar{\Pi}_o(y) + \bar{\Pi}_{os}(y). \tag{10}$$

Employing Eq. (6b), Eq. (9b) may be decomposed in the form

$$\bar{\Pi}_{os}(y) = \sum_1^R \bar{\Pi}_{or}(y), \quad \bar{\Pi}_{or}(y) = \bar{\Pi}_r(y) + \bar{\Pi}_{cr}(y), \tag{11}$$

where by definition

$$\bar{\Pi}_r(y) = \eta_r \bar{E}_r(y), \quad \bar{\Pi}_{cr}(y) = \eta_{cr} \bar{E}_{cr}(y). \tag{12}$$

Again, the loss factors η_r and η_{cr} are the stiffness control loss factors associated with the springs on the fore and the back sides of the mass m_r of the rth satellite oscillator. The spring on the back side constitutes the stiffness element in the coupling form. The spring on the fore side renders the satellite oscillator an *oscillator* rather than merely a *mass*. It is assumed that the damping is provided in these springs only [1,2]. Provisions for other types of damping can be made, but the increase in algebraic complexity can hardly be justified at this stage.

From Eqs. (9b), (11) and (12) one obtains the energetic version of the induced loss factor $\eta_I^e(y)$ in the form

$$\eta_I^e(y) = \sum_1^R \eta_{Ir}^e(y), \quad \eta_{Ir}^e(y) = (y^2/2)\{\eta_r \bar{E}_r(y) + \eta_{cr} \bar{E}_{cr}(y)\}, \tag{13}$$

where $\bar{E}_r(y)$ and $\bar{E}_{cr}(y)$ are more explicitly expressed in Eqs. (4b) and (4c). The induced loss factor $\eta_I(y)$, derived via LIA, is explicitly expressed in Eq. (7c) and is extensively investigated in Refs. [1,2]. To what degree then is $\eta_I^e(y)$ stated in Eq. (13) equal to $\eta_I(y)$? To establish analytically the answer to this question may require undue algebraic manipulations which, again, can hardly be justified at this stage. Instead, the equivalence is cursorily tested computationally. The computations with respect to $\eta_I^e(y)$ are guided by computations performed in Ref. [2]. Again, as in Ref. [2], it is convenient, with only a slight loss in generality, to impose a similarity on the stiffnesses (springs) that are placed on either side of the mass m_r of the rth satellite oscillator; namely

$$(x_r)^2 = \alpha_r (x_r^o)^2, \quad (x_{cr})^2 = \alpha_{cr} (x_r^o)^2, \tag{14a}$$

where x_r^o defines a designed distribution for the normalized resonance frequencies of the satellite oscillators and α_r and α_{cr} are the spring factors. In that design the resonance frequencies ascend according to the value of the index r, i.e., $(x_{rr})^2 \leqslant (x_{qq})^2$, $\quad q = (r+1)$, $1 \leqslant r \leqslant (R-1)$, and the numbers of resonance frequencies on either side of the resonance frequency ω_o of the master oscillator in isolation, are equal. Once again as in Refs. [1,2], x_r^o is assigned the specific form

$$x_r^o = [1 + \{(1+R) - 2r\}(\gamma/2R)]^{-1/2}, \quad \gamma < 1, \quad z_r^o = (x_r^o/y) \tag{14b}$$

and is graphically depicted in Fig. 2a of Ref. [2]. In the same vein, the loss factors η_r and η_{cr} that are assigned to the satellite oscillators and to their couplings are, again, conveniently designated in terms of the modal overlap parameters b_r and b_{cr}, respectively, namely

$$\begin{gathered}\eta_r = (b_r/y)[v_r(y)\omega_o]^{-1}, \quad \eta_{cr} = (b_{cr}/y)[v_r(y)\omega_o]^{-1}, \\ \eta_{rr}(x_{rr})^2 = \eta_r (x_r)^2 + \eta_{cr}(x_{cr})^2 \Rightarrow [(\eta_r \alpha_r) + (\eta_{cr}\alpha_{cr})](x_r^o)^2,\end{gathered} \tag{14c}$$

where $v_r(y)$ is the modal density of the satellite oscillators and Eq. (5) is used [1,3]. Copying the conditions in the computations performed in Ref. [2] these parameters, as well as the mass elements and the parameters that specify the couplings of the satellite oscillators, are assumed to be independent of the index that identifies a satellite oscillator; namely

$$b_r = b_{cr} = b, \quad \bar{m}_r = \bar{m}, \ \ \bar{m}_{cr} = \bar{m}_c, \quad \alpha_r = \alpha, \ \ \alpha_{cr} = \alpha_c, \quad g_r = g. \tag{15}$$

Under these impositions and from Eq. (14b), Eq. (14c) reduces to

$$\eta_r = \eta_{cr} \Rightarrow \eta(y) = (b/y)[v_s(y)\omega_o]^{-1}, \quad [v_s(y)\omega_o]^{-1} = (\gamma/2R)(y)^3,$$
$$\eta(y) = (b/R)(\gamma/2)(y)^2, \tag{14d}$$

where $\eta(y)$ is the loss factor as assigned to a satellite oscillator with a normalized resonance frequency that lies in the vicinity of the normalized frequency (y). The loss factor η_r, in Eq. (14d), is graphically depicted in Fig. 2(b) of Ref. [2]. Further, with these simplifications in place, straightforward, but tedious, algebraic manipulations of Eqs. (4) and (5) and with the use of Eq. (13), one may derive

$$\eta^e_{Ir}(y) = \eta^e(y)\bar{E}_{or}(y) = (b/R)(\gamma/2)(y^4/2)E_{or}(y), \tag{16a}$$

where

$$\begin{aligned}\bar{E}_{or}(y) = {} & \bar{m}(1+\bar{m}_c)^{-2}\{[1-(z^o_r)^2]^2 + [(z^o_r)^2\eta(y)]^2\}^{-1}\\ & \times \{[1+\alpha(z^o_r)^2][\{\bar{m}_c + \alpha_c(z^o_r)^2\}^2 + \{\alpha_c\eta(y)(z^o_r)^2 - (g/y)\}^2]\\ & + \bar{m}_c[\{1-(1+\bar{m}_c+\alpha_c)(z^o_r)^2\}^2 + \{t_+(y)\}^2]\\ & + [\alpha_c(z^o_r)^2][\{(1+2\bar{m}_c) - \alpha(z^o_r)^2\}^2 + \{t_-(y)\}^2]\\ & - (\bar{g}/y)[\{1-(1+\bar{m}_c+\alpha_c)(z^o_r)^2\}\{\alpha\eta(y)(z^o_r)^2 + (g/y)\}\\ & - \{(1+2\bar{m}_c) - \alpha(z^o_r)^2\}\{(1+\bar{m}_c+\alpha_c)\eta(y) - (g/y)\}]\}\end{aligned} \tag{16b}$$

and the explicit expressions for $t_+(y)$ and $t_-(y)$ are

$$t_+(y) = (1+\bar{m}_c)\eta(y)(z^o_r)^2 + [\alpha_c\eta(y)(z^o_r)^2 - (g/y)],$$
$$t_-(y) = (1+\bar{m}_c)\eta(y)(z^o_r)^2 - [\alpha_c\eta(y)(z^o_r)^2 - (g/y)]. \tag{16c}$$

Using Eq. (16) the energetically determined induced loss factor $\eta^e_I(y)$ is computed, as a function of the normalized frequency (y), and the results are recorded. Analogously to Ref. [2], these computations are carried out assigning the standard values

$$\bar{m}_c = g = 0, \quad \alpha_c = 1, \quad (M_s/M_o) = 10^{-1}, \quad b = 0.1, \quad \gamma = 0.6 \text{ and } R = 27, \tag{17}$$

where $\bar{m}_c$, g and α_c define the couplings, γ is the frequency bandwidth, $M_s = Rm$ defines the global mass of the satellite oscillators and R is the number of satellite oscillators in the set. When these standard assignments are deviated from, specific mentions are to be rendered, notwithstanding that, at times, the employment of these standard values may be reiterated. It transpires that the computed results, under the standard value and the several variations on these values, yield levels of $\eta^e_I(y)$ that are identical to those determined in Ref. [2] for the induced loss factor $\eta_I(y)$. For this reason, there is no need to repeat the results of these computations; the results presented in Figs. 3–5 of this reference suffice. One may inquire: what about the results concerning the mean-value levels of the induced loss factor $\langle\eta_I(y)\rangle$, how do they compare with the mean-value levels of the energetically determined induced loss factor $\langle\eta^e_I(y)\rangle$? Repeating the procedure, proposed in Refs. [1,2], the mean-value levels $\langle\eta^e_I(y)\rangle$ of this induced loss factor are determined:

$$\langle\eta^e_I(y)\rangle = \sum_1^R (R')^{-1} \sum_{r'=(-R'/2)}^{(R'/2)} \eta^e_I(y, r + (r'/R')] \Rightarrow \int_{(1/2)}^{R+(1/2)} \eta^e_I(y, r)\,\mathrm{d}r. \tag{18}$$

Rendering the index r continuous in Eq. (16) and substituting the result in Eq. (18), one evaluates $\langle \eta_I^e(y) \rangle$ by carrying out the integration [2]. This evaluation yields

$$\langle \eta_I^e(y) \rangle = D[C(y) + O^e\{\eta^e(y)\}^2], \quad \eta_I^{e1}(y) = DC(y), \tag{19}$$

where (D), $C(y)$, O^e and the range of validity in y are

$$D = [(\pi/2)(2/\gamma)(M_s/M_o)], \quad M_s = (Rm), \tag{20a}$$

$$C = [(\bar{m}_c + \alpha_c)^2 + (g/y)^2][1 + \bar{m}_c]^{-1}, \quad (1 + \bar{m}_c) = (\alpha + \alpha_c), \tag{20b}$$

$$O^e = (1/2)[(1 + \bar{m}_c)(m_c + \alpha_c) + 2\bar{m}_c\alpha_c], \tag{20c}$$

$$[1 + (\gamma/2)]^{-(1/2)} \leqslant y \leqslant [1 - (\gamma/2)]^{-(1/2)}, \quad \Delta(y) \simeq (\gamma/2), \tag{20d}$$

respectively. Again, analogous to the definition in Ref. [2], $\eta_I^{e1}(y)$ is the first order approximation (FOA) of $\langle \eta_I^e(y) \rangle$. From a consultation with Ref. [2], one finds that not only is $\eta_I(y) = \eta_I^e(y)$, but also $\eta_I^1(y) = \eta_I^{e1}$, notwithstanding that the higher approximations in the two cases, O versus O^e, are rather different. As argued previously, this difference lies outside the scope of Ref. [2] as well as of this paper. Clearly, since the FOA $\eta_I^1(y)$ is depicted in Figs. 3–5 of Ref. [2], this depiction, as do the rest of the curves in these figures, pertain in full to the energetically determined forms of the induced loss factor. With this equivalence in mind, the focus is shifted toward matters that pertain largely to the EA and transcend the equivalence just mentioned.

4. Renormalization of stored energies and powers—illegitimate and legitimate loss factors

Quantities, e.g., the stored energy $E_{os}(y)$ and the dissipated power $\Pi_{os}(y)$, when normalized by $E_{oK}(y)$ and $\{\omega E_{oK}(y)\}$, respectively, are designated by a single bar; namely, $\bar{E}_{os}(y)$ and $\bar{\Pi}_{os}(y)$. Situations arise in which a normalization by $E_o(y)$ and $\{\omega E_o(y)\}$, respectively, may be preferred. Such normalizations are to be designated by a *double bar*; namely, $\bar{\bar{E}}_{os}(y)$ and $\bar{\bar{\Pi}}_{os}(y)$, respectively. In particular then,

$$\eta_\infty(y) = [\Pi_o(y)/\{\omega E_o(y)\}] = \bar{\bar{\Pi}}_o(y) = 2[1 + (y)^2]^{-1}\eta_{o,} \tag{21a}$$

$$\eta_{oI}^e(y) = [\Pi_{os}(y)/\{\omega E_o(y)\}] = \bar{\bar{\Pi}}_{os}(y) = 2[1 + (y)^2]^{-1}\eta_I^e(y), \tag{21b}$$

$$\eta_{oI}^e(y) = \sum_1^R \eta_r(r)\bar{\bar{E}}_{or}(y) = \eta^e(y)\bar{\bar{E}}_{os}(y),$$

$$\eta_{oI}^e(y) = 2[1 + (y)^2]^{-1}\eta_I^e(y) = [\eta^e(y)\Xi_o^s(y)], \tag{21c}$$

$$\Xi_o^s(y) = \bar{\bar{E}}_{os}(y) = [E_{os}(y)/E_o(y)], \quad \eta^e(y) = 2[1 + (y)^2]^{-1}\eta(y). \tag{21d}$$

It emerges then that whereas the loss factor η_o assigned to the master oscillator and the loss factor η_I^e induced in the master oscillator by a set of satellite oscillators are governed by the normalization employing the kinetic energy $E_{oK}(y)$ and the power $\{\omega E_{oK}(y)\}$, the loss factors $\eta_\infty(y)$ and $\eta_{oI}^e(y)$ are governed, correspondingly, by quantities normalized by the stored energy $E_o(y)$ and the power $\{\omega E_o(y)\}$. The stored energy $E_o(y)$ is the combined energy, consisting of the kinetic energy $E_{oK}(y)$ and the potential energy $E_{oP}(y)$ stored in the master oscillator, $E_o(y) = E_{oK}(y) + E_{oP}(y)$, as stated in Eq. (1). Eq. (21) is just a renormalized version of Eq. (13). The ratio of the energy $E_{os}(y)$ stored in the satellite oscillators and in the couplings to that of the energy $E_o(y)$ stored in the master oscillator is dubbed the *global coupling strength* and, as such, is designated $\Xi_o^s(y)$. Related to this quantity is the modal coupling strength $\zeta_o^s(y)$; the relationship is

$$\Xi_o^s(y) = [v_s(y)/v_o(y)]\zeta_o^s, \quad R = \Delta(y)[v_s(y)\omega_o], \quad N_o = \Delta(y)[v_o(y)\omega_o], \tag{22}$$

where $v_o(y)$ and $v_s(y)$ are the modal densities in the master and in the adjunct dynamic systems, respectively. The significance of Eq. (22) will subsequently emerge. At this stage a minor digression may be in order: Were one to construct, from Eqs. (19) and (21c), the hybrid expression

$$[\eta^e(y)\Xi_o^s(y)]_h = 2[1+y^2]^{-1}\begin{cases} \eta_I^{e1}(y), & b \leqslant 1, \\ \eta_I^e(y), & b \geqslant 1, \end{cases} \tag{23a, b}$$

then, except for erosion at the higher values of the modal overlap parameter, $b > 1$, one finds the hybrid quantity $[\ldots]_h$ to be largely independent of b [1,2]. Referring to Eq. (22), it follows that in the hybrid milieu $\eta^e(y)$ is inversely proportional to $\Xi_o^s(y)$ and vice versa. Taking note of Eq. (19), there is no way for a physically acceptable complex to entertain a vanishing $\eta^e(y)$. Of course, Eq. (23) does not fall under this spell, but neither can one ignore the undulations that beset the energetically determined induced loss factor $\eta_I^e(y)$ when b is reduced with the intention of rendering $\eta^e(y)$ negligible [1,2].

Being cognizant of the second of Eq. (21d), the global coupling strength $\Xi_o^s(y)$ may feature in another renormalization in which the entire energy $E_e(y)$ stored in the complex is employed. This stored energy is given by

$$E_e(y) = E_o(y) + E_{os}(y) = E_o(y)[1 + \Xi_o^s(y)] \tag{24}$$

and the corresponding normalization for the powers is given by $\{\omega E_e(y)\}$. Such renormalizations are to be designated by a *tilde* over the quantity, e.g.,

$$\tilde{E}_{os}(y) = [E_{os}(y)/E_e(y)], \quad \tilde{\Pi}_{os} = [\Pi_{os}(y)/\{\omega E_e(y)\}]. \tag{25a}$$

From Eqs. (21), (24) and (25a) one may derive

$$\tilde{E}_{os}(y) = [E_{os}(y)/E_e(y)] = \bar{E}_{os}(y)[1 + \Xi_o^s(y)]^{-1} \tag{25b}$$

$$\tilde{\Pi}_{os}(y) = [\Pi_{os}(y)/\{\omega E_e(y)\}] = \bar{\bar{\Pi}}_{os}(y)[1 + \Xi_o^s(y)]^{-1}, \tag{25c}$$

$$\bar{\bar{E}}_e(y) = [1 + \Xi_o^s(y)]. \tag{25d}$$

The two renormalizations just introduced, in Eqs. (21) and (25), lead to the definition of two distinct loss factors. The first is defined in terms of the energy $E_o(y)$ stored in the master oscillator only, namely

$$\eta_t^e(y) = [\Pi_e(y)/\{\omega E_o(y)\}] = [\eta_\infty(y) + \eta^e(y)\Xi_o^s(y)], \tag{26a}$$

where use is made of Eqs. (8) and (10). The second is defined in terms of the energy $E_e(y)$ stored in the entire complex, namely

$$\eta_e^e(y) = \Pi_e(y)/\{\omega E_e(y)\} = \eta_t^e(y)[1 + \Xi_o^s(y)]^{-1}, \tag{27a}$$

where use is made of Eqs. (25) and (26a). These two loss factors were previously defined and discussed in Refs. [5,6]. In these discussions it was claimed that $\eta_t^e(y)$ is not a legitimate loss factor. This designation was primarily predicated on assigning to the definition of this loss factor the entire external input power $\Pi_e(y)$, but accounting for the stored energy $E_o(y)$ in the master oscillator only, thereby ignoring the portion of the stored energy that is maintained in the satellite oscillators and in the couplings. (No wonder the question of "where did the energy go?" arose from accepting $\eta_t^e(y)$ as a legitimate loss factor [6–11].) In this connection one recalls, from Eqs. (21) and (24), that

$$\eta_t^e(y) = [\eta_\infty(y) + \eta_{oI}^e(y)], \quad \eta_{oI}^e(y) = [\eta^e(y)\Xi_o^s(y)]. \tag{26b}$$

Again, since $\eta_{oI}^e(y)$ is found to be largely independent of $\eta^e(y)$, one concludes, from Eq. (26b), that $\eta_t^e(y)$ is also independent of $\eta^e(y)$. Eqs. (26a) and (26b) makes clear that to change $\eta_t^e(y)$ either the coupling parameters, the mass ratio (M_s/M_o), or both need changing. In this assessment, η_o is assumed to be fixed, and $\eta_\infty(y) = 2\eta_o(1+y^2)^{-1}$ [cf., Eq. (21a)]. On the other hand, the second loss factor, dubbed the *effective* loss factor and designated $\eta_e^e(y)$, does take into account the whole energy stored in the complex [5,6]. It may thus be designated a legitimate loss factor. From Eqs. (24) and (27a) one finds

$$\eta_e^e(y) = \eta_t^e(y)\{\eta^e(y)[\eta^e(y) + \eta_{oI}^e(y)]^{-1}\} \leqslant \eta_t^e(y). \tag{27b}$$

In particular, if the induced loss factor $\eta_{oI}^e(y)$ exceeds the indigenous loss factor $\eta_\infty(y)$ of the master oscillator, Eq. (27b) may be reduced to

$$\eta_e^e(y) \cong [\eta^e(y)\eta_{oI}^e(y)][\eta^e(y) + \eta_{oI}^e(y)]^{-1} < \begin{cases} \eta_{oI}^e \\ \eta^e(y) \end{cases}, \quad \eta_\infty(y) < \eta_{oI}^o(y). \tag{27c}$$

Thus, the effective loss factor $\eta_e^e(y)$, under this condition, is a parallel combination of the induced loss factor $\eta_{oI}^e(y)$ and the loss factor $\eta^e(y)$ assigned to a typical satellite oscillator. It follows that $\eta_e^e(y)$ is less than either one of these loss factors, as stated in Eq. (27c). Consider a reasonable complex for which $\eta_{oI}^e(y) > \eta^e(y)$. Under this additional condition, Eq. (27c) may be reduced to

$$\eta_e^e(y) \cong \eta^e(y), \quad \eta_{oI}(y) \gg \eta^e(y), \Xi_o^s(y) \gg 1, \quad \eta_e^e \ll \eta_t^e(y). \tag{28}$$

The loss factors $\eta_t^e(y)$ and $\eta_e^e(y)$, as functions of y, are evaluated and contrasted. Again, only a few representative cases are displayed. The displays are given in Figs. 2–4. The presentation covers cases that parametrically conform to those governing Figs. 3–5 of Ref [2], respectively. The two loss factors, $\eta_t^e(y)$ and $\eta_e^e(y)$, are contrasted on the same figure and each is compounded by a superimposition of curves that pertain, in turn, to modal overlap parameters $b =$ (0.1), (2.0) and (10). The disparity between $\eta_t^e(y)$ and $\eta_e^e(y)$ is significantly high for the lower values of the modal coupling overlap (b); at the higher values of b, the disparity largely dissipates. It is also observed that the disparity is dependent not only on the coupling strength, but also slightly on the number of satellite oscillators in the set; e.g., compare Figs. 2 and 3.

In addition to the evaluations of $\eta_t^e(y)$and $\eta_e^e(y)$ the corresponding levels of $[(R)^{-1}\Xi_o^s(y)]$ are also evaluated, as a function of y, and are displayed in Figs. 5–7. Since in the complex under investigation, $N_o = 1$, Eq. (22) defines the quantity $[(R)]^{-1}\Xi_o^s(y)]$ as equal to the modal coupling strength $\zeta_o^s(y)$. Again, three curves that pertain to the modal overlap parameters $b =$ (0.1), (2.0)

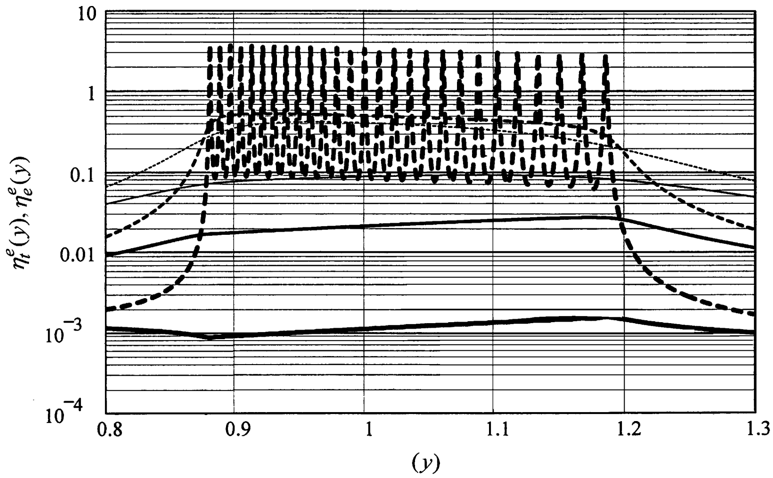

Fig. 2. Loss factors $\eta_t^e(y)$: ▬ ▬ ▬, $b = 0.1$; - - - -, 2.0; - - - - -, 10; and $\eta_e^e(y)$: ▬▬, $b = 0.1$; ——, 2.0; ——, 10; as functions of y, with stiffness control couplings. $R = 27$, $M_s/M_o = 0.1$ and $\eta_o = (10^{-3})$. Sprung masses: $\alpha_c = 1.0$ $(\alpha = 0.0)$, $g = \bar{m}_c = 0$ (strong coupling).

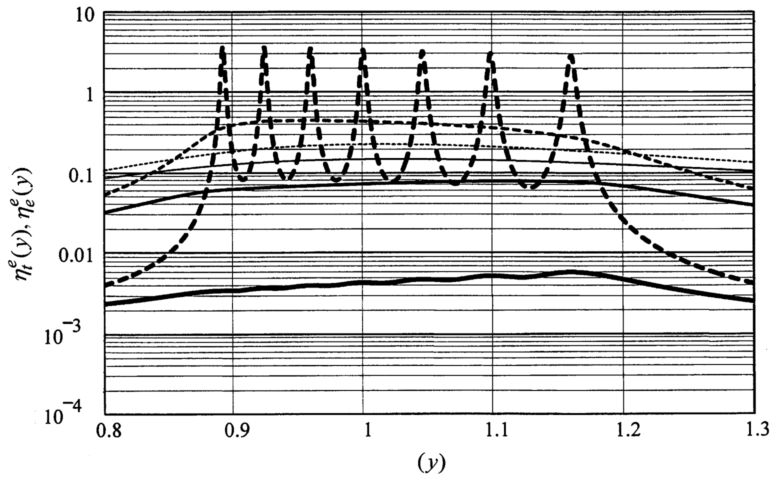

Fig. 3. Same as Fig. 2, except that R is changed from 27 to 7.

and (10) are superimposed in these figures. The undulations in the levels when b is less than unity and the suppression of the undulations when b is in excess of unity is clearly apparent in these figures. The first order approximation (FOA) of $\zeta_o^s(y)$ are also superimposed on these figures. This superimposition exposes, once again, the phenomenon that the mean-value averaging of the undulations coincide with the FOA and that the phenomenon of erosion commences and increases as b approaches and increases beyond unity [1,2,5,10]. A significant feature, nevertheless, is revealed in Figs. 5–7: contrary to its value in the SEA the modal coupling strength $\zeta_o^s(y)$ in the

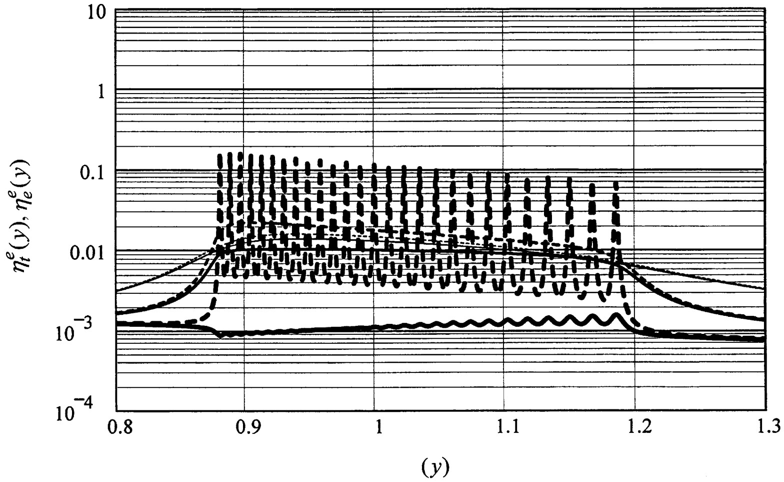

Fig. 4. Loss Factors $\eta_t^e(y)$ (dashed) and $\eta_e^e(y)$ (solid), as functions of y, with gyroscopic control couplings. $R = 27$, $M_s/M_o = 0.1$ and $\eta_o = (10^{-3})$. Shown are curves for three values of the modal overlap parameter (b): ▬ ▬ ▬, $b = 0.1$; - - - -, 2.0; - - - - -, 10; ▬▬, $b = 0.1$; ——, 2.0; ——, 10; $\alpha_c = \bar{m}_c = 0$ ($\alpha = 1.0$), $g = 0.15$ (moderate coupling).

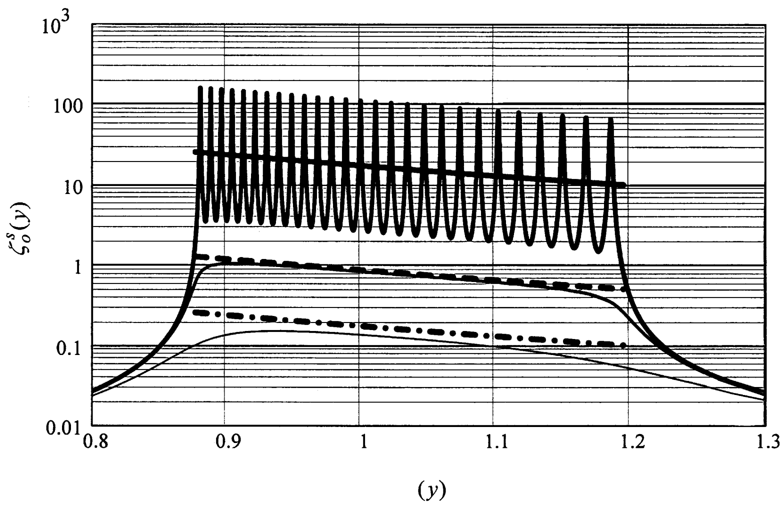

Fig. 5. Modal coupling strength $\zeta_o^s(y)$, as a function of y, for sprung masses: $\alpha_c = 1.0$ ($\alpha = 0.0.$), $g = m_c = 0$ (strong coupling). Shown are curves pertaining to three values of the modal overlap parameter (b): ▬▬, $b = 0.1$; ——, 2.0; ——, 10; and the first order approximation (FOA): ▬▬, $b = 0.1$; ▬ ▬, 2.0; ▬ • ▬; 10. $R = 27$, $M_s/M_o = 0.1$.

EA may exceed unity. Indeed, when the coupling is strong, as it is in Figs. 5 and 6, and b is an order of magnitude less than unity, $\zeta_o^s(y)$ substantially exceeds unity. Moreover, even for moderate coupling, as it is in Fig. 7, and $b = 0.1$, $\zeta_o^s(y)$ is close to unity. Only for weak coupling and/or for b that exceeds unity, is $\zeta_o^s(y)$ much less than unity. The significance of this feature is examined next.

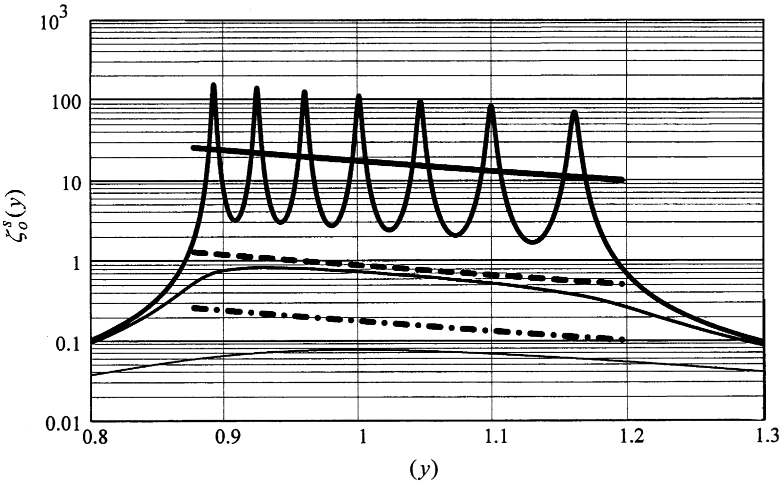

Fig. 6. Same as Fig. 5, except that R is changed from 27 to 7.

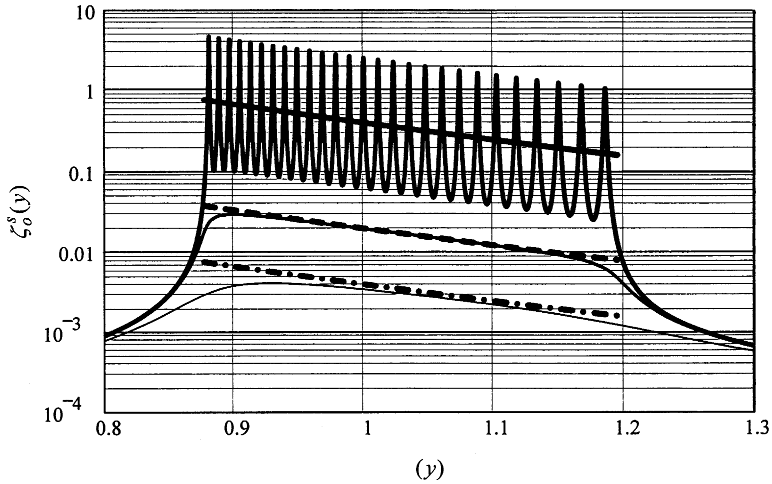

Fig. 7. Modal coupling strength $\zeta_o^s(y)$, as a function of y, for gyroscopic control couplings: $\alpha_c = \bar{m}_c = 0$ ($\alpha = 1.0.$), $g = 0.15$ (moderate coupling). Shown are curves pertaining to three values of the modal overlap parameter (b): ——, $b = 0.1$; ——, 2.0; ——, 10 and the first order approximations (FOA): ——, $b = 0.1$; ▬ ▬, 2.0; ▬ • ▬; 10. $R = 27$, $M_s/M_o = 0.1$.

5. Relationship to the SEA

It may be conducive to include in this paper a possible relationship between the EA, dealt with in the preceding two sections, and the SEA. The latter analysis was initiated in the early 1960s at BBN and subsequently has become a major tool in noise control engineering [3,15,16]. A rudimentary SEA is applied to a complex comprising of a master oscillator coupled to a set of individual satellite oscillators; the satellite oscillators are neither coupled to each other nor externally driven [cf., Fig. 1]. In terms of SEA the equations that govern the energy distribution in the complex are

$$[\eta_{\infty}(y) + \sum_{1}^{R} \eta_{ro}(y)]E_o(y) - \sum_{1}^{R} \eta_{or}(y)E_{or}(y) = [\Pi_e(y)/\omega], \tag{29a}$$

$$[\eta_r(y) + \eta_{or}(y)]E_{or}(y) - \eta_{ro}(y)E_o(y) = 0, \tag{29b}$$

where $E_o(y)$, $E_{or}(y)$, $\eta_{\infty}(y)$, $\eta_r(y)$ and $\Pi_e(y)$ are previously defined with respect to Eqs. (1)–(3), (6b), (8), (14c) and (21a) [3,16]. In Eq. (29), $\eta_{or}(y)$ and $\eta_{ro}(y)$ are the *coupling loss factors* from the rth satellite oscillator to the master oscillator and vice versa, respectively. In conservative couplings, $\eta_{or}(y) = \eta_{ro}(y)$ [3]. After a straightforward algebraic manipulation of Eq. (29) one obtains

$$\eta_t^e(y) = [\eta_{\infty}(y) + \eta_{oI}^e(y)], \quad \eta_{oI}^e(y) = \sum_{1}^{R} \eta_r(y)\zeta_o^r(y),$$
$$\zeta_o^r(y) = [E_{or}(y)/E_o(y)] = \eta_{ro}(y)[\eta_r(y) + \eta_{or}(y)]^{-1}, \tag{30}$$

where Eqs. (26a) and (26b) is consulted and $\zeta_o^r(y)$ is the *modal coupling strength* of the rth satellite oscillator to the master oscillator [3,16]. With a *sleight-of-statistical hand*, Eq. (30) may be converted to read

$$\eta_{oI}^e(y) = \eta^e(y)\Xi^{SEA}(y), \quad \Xi_o^{SEA}(y) \simeq \sum_{1}^{R} \zeta_o^r(y) = R\zeta_o^{SEA}(y),$$
$$\zeta_o^{SEA}(y) \simeq \eta_{os}^e(y)[\eta^e(y) + \eta_{os}^e(y)]^{-1} < 1, \tag{31}$$

where $\eta^e(y)$ and $\eta_{os}^e(y)$ are a typical loss factor and a typical coupling loss factor to the master oscillator, respectively, of a satellite oscillator with normalized resonance frequency that lies in the vicinity of the normalized frequency (y). The superscript (SEA) designates quantities derived on the basis of SEA [3]. It is a tenet of the SEA that the model coupling strength $\zeta_o^{SEA}(y)$ is less than unity, as is emphasized in Eq. (31).

From Eq. (14d), (21) and (22) the modal coupling strength $\zeta_o^r(y)$ derived via the EA is given by

$$\zeta_o^s(y) = \eta_I^e(y)[R\eta(y)]^{-1} = \eta_I^e(y)[(b\gamma/2)(y)^2]^{-1}. \tag{32a}$$

Taking mean-value levels *ala* Skudrzyk on both sides of Eq. (32a), one obtains

$$\langle \zeta_o^s(y) \rangle = \langle \eta_I^e(y) \rangle [(b\gamma/2)(y)^2]^{-1}. \tag{32b}$$

Using Eq. (19), the FOA $\zeta_o^{s1}(y)$ for the modal coupling strength becomes

$$\zeta_o^{s1}(y) = \eta_I^{e1}(y)[(b\gamma/2)(y)^2]^{-1} = D[(\gamma/2)y^2]^{-1}[C(y)/b],$$
$$D = (\pi/2)(2/\gamma)(M_s/M_o), \quad C(y) = [(\bar{m}_c + \alpha_c)^2 + (g/y)^2][1 + \bar{m}_c]^{-1} \tag{33a}$$

and hence

$$\zeta_o^{s1} = (\pi/2)[2/(\gamma y)]^2(M_s/M_o)[C(y)/b)]. \tag{33b}$$

For the EA, appropriately averaged, to be compatible with the SEA, $\zeta_o^{s1}(y)$ needs to be necessarily maintained below unity. Imposing this necessary condition on Eq. (33) requires b to exceed the

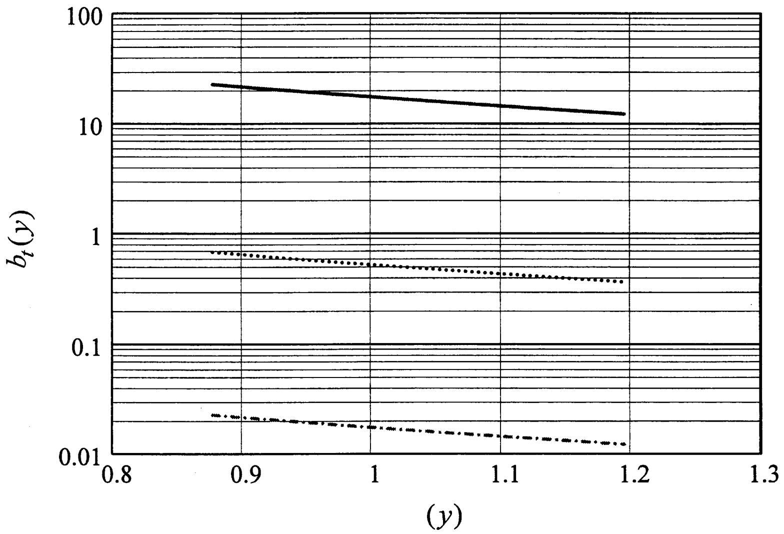

Fig. 8. The threshold $b_t(y)$ of the modal overlap parameter, as a function of the normalized frequency y, for three values of the coupling factor: ——, $C(y) = 1.0$; • • • • •, $C(y) = 3 \times 10^{-2}$; • – • – •, $C(y) = 10^{-3}$; cf., Eq. (20b). $R = 27$, $M_s/M_o = 0.1$.

threshold value $b_t(y)$: $b > b_t(y)$. The condition, by definition, extends only over the normalized frequency range that spans the normalized resonance frequencies of the satellite oscillators. From Eq. (33) one obtains, for the complex under investigation, the expression for the threshold in the form

$$b_t(y) \simeq (\pi/2)[2/(\gamma y)^2](M_s/M_o)[C(y)],$$
$$[1 + (\gamma/2]^{-1/2} \leqslant y \leqslant [1 - (\gamma/2)]^{-1/2}. \tag{34}$$

It thus emerges that SEA has a modal overlap parameter threshold; a situation arises in which SEA is not valid for certain degrees of coupling strengths unless the modal overlap parameter exceeds that threshold. In Fig. 8, $b_t(y)$ is depicted as a function of y, for the standard values of γ and M_s/M_o stated in Eq. (17) and for three values of $C(y)$: 1.0, 3×10^{-2} and 10^{-3} [2]. One finds, for example, that when the coupling is strong, $C(y) = 1$, as is the case for sprung masses, the threshold $b_t(y)$ may exceed unity. Even for moderate coupling, $C(y) = 3 \times 10^{-2}$, the threshold values shown in Fig. 8 is not far below unity. Only for weak coupling, $C(y) = 10^{-3}$, is the value of the threshold far below unity. Of course, in this last situation the modal coupling strength $\zeta_o^s(y)$ undulates. These undulations are, by definition, suppressed in the corresponding first order approximation (FOA). One is reminded that this quantity governs Fig. 8 and Eq. (34) and, therefore, no undulations in levels appear [cf., Figs. 5–7 and Eq. (23)].

References

[1] G. Maidanik, Induced damping by a nearly continuous distribution of nearly undamped oscillators: linear analysis, Journal of Sound and Vibration 240 (2000) 717–731.

[2] G. Maidanik, K.J. Becker, Dependence of the induced damping on the coupling forms and coupling strengths: linear analysis, Journal of Sound and Vibration 266 (2003) 15–32, this issue.
[3] R.H. Lyon, Statistical Energy Analysis of Dynamic Systems; Theory and Applications, MIT, Cambridge, 1975; R.H. Lyon, R.G. Dejung, Theory and Application of Statistical Energy Analysis, Butterworth-Heinemann, Boston, 1995.
[4] R.H. Lyon, G. Maidanik, Power flow between linearly coupled oscillators, Journal of the Acoustical Society of America 34 (1962) 623–639.
[5] G. Maidanik, K.J. Becker, Various loss factors of a master harmonic oscillator that is coupled to a number of satellite harmonic oscillators, Journal of the Acoustical Society of America 103 (1998) 3184–3195.
[6] G. Maidanik, Power dissipation in a sprung mass attached to a master structure, Journal of the Acoustical Society of America 98 (1995) 3527–3533.
[7] A. Pierce, V.W. Sparrow, D.A. Russell, Fundamental structural-acoustic idealizations for structures with fuzzy internals, Journal of Acoustics and Vibration 117 (1995) 339–348.
[8] M. Strasberg, D. Feit, Vibration damping of large structures by attached small resonant structures, Journal of the Acoustical Society of America 99 (1996) 335–344.
[9] R.L. Weaver, Mean and mean-square responses of a prototypical master/fuzzy structure, Journal of the Acoustical Society of America 99 (1996) 2528–2529.
[10] M.J. Brennan, Wideband vibration neutralizer, Noise Control Engineering Journal 45 (1997) 201–207.
[11] R.J. Nagem, I. Veljkovic, G. Sandri, Vibration damping by a continuous distribution of undamped oscillator, Journal of Sound and Vibration 207 (1997) 429–434.
[12] Yu.A. Kobelev, Absorption of sound waves in a thin layer, Soviet Physics Acoustics 33 (1987) 295–296.
[13] G. Maidanik, K.J. Becker, Noise control of a master harmonic oscillator coupled to a set of satellite harmonic oscillators, Journal of the Acoustical Society of America 104 (1998) 2628–2637;
G. Maidanik, K.J. Becker, Characteristics of multiple-sprung mass for wideband noise control, Journal of the Acoustical Society of America 106 (1999) 3119–3127.
[14] E. Skudrzyk, The mean-value method of predicting the dynamic response of complex vibrators, Journal of the Acoustical Society of America 67 (1980) 1105–1135.
[15] Proceedings of the Symposium of Statistical Energy Analysis 1997 IUTAM, Southampton, England and Proceedings of the First International AutoSEA Users Conference, 2000 July 27–28, Vibro-Acoustic Sciences, 12555 High Bluff Drive, Suite 310, San Diego, CA 92130.
[16] G. Maidanik, J. Dickey, On the external input power into coupled structures, Proceedings of the Symposium of Statistical Energy Analysis, IUTAM, Southampton, England, 1997.